

Enhanced damping of ion acoustic waves in dense plasmas

S. Son*

18 Caleb Lane, Princeton, NJ 08540

Sung Joon Moon†

Program in Applied and Computational Mathematics, Princeton University, Princeton, NJ 08544

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A theory for the ion acoustic wave damping in dense plasmas and warm dense matter, accounting for the Umklapp process, is presented. A higher decay rate compared to the prediction from the Landau damping theory is predicted for high-Z dense plasmas where the electron density ranges from 10^{21} to 10^{24} cm $^{-3}$ and the electron temperature is moderately higher than the Fermi energy.

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The ion acoustic wave, a longitudinal collective mode in plasmas, plays a crucial role in a range of applications, such as the Thomson scattering [1, 2] and the Brillouin scattering [3]. Understanding the dynamics of the wave in dense plasmas or warm dense matter is important in various context, including the inertial confinement fusion [4, 5] and the compression of x-rays [6–9].

The decay rate of the ion acoustic waves in plasmas is often modeled by the prevalent Landau damping theory. However, this theory is inadequate for *dense* plasmas, as the Umklapp process, which is not accounted for, becomes pronounced in high densities. It was shown that the Umklapp process dominates the Landau damping for low- k plasmons [10–12]. Even though the detailed underlying physical mechanisms of the plasmons are different from those of the ion acoustic waves, it is expected that the Umklapp process is also important to the ion acoustic waves. The goal of this paper is to estimate the effect of this process. Starting from the plasmon damping theory in dense plasmas [11], a new theory predicting the ion acoustic wave sampling is proposed and a regime where the decay rate is larger than the prediction by the Landau damping theory is identified. This result would have implications on the Brillouin scattering of dense plasmas, the x-ray Thomson scattering, and the reflection problem in the inertial confinement fusion.

First we provide a brief review on the Landau damping theory for the ion-acoustic wave. Only a neutral plasma of single ion-species ions is considered for simplicity. We denote the electron (ion) temperature by T_e (T_i), the corresponding density by n_e (n_i), the mass by m_e (m_i), and the charge by $Z_e = 1$ ($Z_i = Z$), where the charge neutrality condition reads $n_e = Zn_i$. The longitudinal dielectric function is $\epsilon(\mathbf{k}, \omega) = 1 + \chi_e + \chi_i$, where

$$\chi_{i,e} = \frac{\omega_{i,e}^2}{k^2} \int \frac{\mathbf{k} \cdot \nabla_{\mathbf{v}} f_{i,e}}{\omega - \mathbf{k} \cdot \mathbf{v}} d^3\mathbf{v}, \quad (1)$$

$\omega_{i,e}^2 = 4\pi n_{i,e} Z_{i,e} e^2 / m_{i,e}$ is the ion (electron) plasmon

frequency, and $f_{i,e}$ is normalized as $\int f_{i,e} = 1 d^3\mathbf{v}$. We assume $v_{ti} < \omega/k \ll v_{te}$, where $v_{ti,te} = \sqrt{T_{i,e}/m_{i,e}}$. This is a necessary condition for a moderate Landau damping. Under this assumption, χ_e and χ_i can be estimated to be $\chi_e \cong 1/(k\lambda_{de})^2$ and $\chi_i \cong -(\omega_i/\omega)^2$, where $\lambda_{de} = \sqrt{T_e/4\pi n_e e^2}$ is the Debye screening length. The condition $\epsilon = 0$ yields the dispersion relation for the ion acoustic wave,

$$\omega_{iaw}^2 = \frac{k^2 V^2}{1 + (k\lambda_{de})^2}, \quad (2)$$

where $V = \sqrt{ZT_e/m_i}$. The ion acoustic wave decay rate from the Landau damping theory is given to be

$$\gamma/\omega_{iaw} = \left(\frac{\omega_{iaw}}{\omega_i} \right)^2 \text{Im} [\chi_e + \chi_i], \quad (3)$$

where

$$\text{Im} [\chi_{i,e}] = \left(\frac{\omega_{i,e}}{kv_{ti,te}} \right)^2 S \frac{1}{\sqrt{2\pi}} \exp(-S^2/2),$$

and $S = \omega_{iaw}/kv_{te,ti}$.

According to the Landau damping theory for the ion acoustic wave, there are many electrons satisfying the resonance condition, of which energy is very low compared to the average kinetic energy. The distribution function around the resonance condition hardly varies, and nearly even electron population on both sides of the resonance condition results in little net energy transfer between the wave and the electrons. In other words, the derivative of the distribution function with respect to the velocity nearly vanishes at the resonance condition, which makes the decay rate small. This physical picture that electrons are freely-streaming and interacting only with the wave is no longer accurate in dense plasmas, because the distortion in the electron motion due to the presence of the ions (i.e., the Umklapp process) becomes important [10, 11].

The effect of the Umklapp process on the ion acoustic wave damping can be analyzed by an approach similar to what was taken in developing the plasmon damping theory for dense plasmas in Ref. [11]. Below we

†Current Address: 28 Benjamin Rush Ln. Princeton, NJ 08540

*Electronic address: seunghyeonson@gmail.com

follow nearly the same steps as in Sec. IV therein. In the presence of the potential of the form $\phi(\mathbf{x}, t) = \phi \exp(i\mathbf{k} \cdot \mathbf{x} - i\omega t) + \phi^* \exp(-i\mathbf{k} \cdot \mathbf{x} + i\omega t)$, the wave packet is modified to be

$$\begin{aligned} |\sigma\rangle &= |\sigma\rangle + \sum_{\sigma_1} \frac{e\phi}{\hbar\omega - E_{\sigma_1} + E_{\sigma}} |\sigma_1\rangle \langle\sigma_1| \exp(i\mathbf{k} \cdot \mathbf{x}) |\sigma\rangle \\ &+ \sum_{\sigma_1} \frac{e\phi^*}{-\hbar\omega + E_{\sigma_1} - E_{\sigma}} |\sigma_1\rangle \langle\sigma_1| \exp(-i\mathbf{k} \cdot \mathbf{x}) |\sigma\rangle, \end{aligned}$$

where the perturbation theory of the first order is used assuming the perturbation is weak, $|\sigma\rangle$ denotes the original eigenstate, $|\sigma_1\rangle$ denotes the perturbed eigenstate, and σ is an index for the eigenstate. Then the perturbation in the density is given to be

$$\begin{aligned} \delta n(k, \omega) &= \sum_{\sigma_1} f(\sigma_1) |\langle\sigma_1| \exp(i\mathbf{k} \cdot \mathbf{x}) |\sigma_1\rangle|^2 \\ &= \sum_{\sigma_1, \sigma_2} e\phi(f(\sigma_1) - f(\sigma_2)) \beta(\sigma_1, \sigma_2, \mathbf{k}, \omega), \end{aligned}$$

where $f(\sigma_i)$ is the occupation number, and $\beta(\sigma_1, \sigma_2)$ is

$$\beta(\sigma_1, \sigma_2) = \frac{\langle\sigma_1| \exp(-i\mathbf{k} \cdot \mathbf{x}) |\sigma_2\rangle \langle\sigma_2| \exp(i\mathbf{k} \cdot \mathbf{x}) |\sigma_1\rangle}{\hbar\omega - E_{\sigma_1} + E_{\sigma_2}}.$$

χ_e is, up to the first order in ϕ ,

$$\begin{aligned} \chi_e &= \frac{4\pi n_e e^2}{k^2} \sum_{\sigma_1, \sigma_2} e\phi(f(\sigma_1) - f(\sigma_2)) \\ &\times \frac{\langle\sigma_1| \exp(-i\mathbf{k} \cdot \mathbf{x}) |\sigma_2\rangle \langle\sigma_2| \exp(i\mathbf{k} \cdot \mathbf{x}) |\sigma_1\rangle}{\hbar\omega - E_{\sigma_1} + E_{\sigma_2}}. \end{aligned} \quad (4)$$

With an appropriate choice of the eigenstates under a given condition, Eq. (4) can be applied to various situations. For example, Sturm [10] used the eigenstate $|\sigma\rangle = |\mathbf{q}\rangle + \sum_{\mathbf{q}_1 \neq \mathbf{q}} |\mathbf{q}_1\rangle \langle\mathbf{q}_1| V |\mathbf{q}\rangle / (E_{\mathbf{q}} - E_{\mathbf{q}_1})$, where \mathbf{q} and \mathbf{q}_1 are the wave vectors, and $\langle\mathbf{q}_1| V |\mathbf{q}\rangle$ is the pseudo-potential. We will suggest an appropriate eigenstate for the ion acoustic wave, after discussing the ion dynamics below.

Assuming each ion is located at \mathbf{X}_i in the time scale of the electron damping on the wave (i.e., the Born-Oppenheimer approximation), we compute the damping rate of the ion-acoustic wave due to the electrons in the presence of the spatially fixed ions, and then obtain the average dynamics by integrating over the probability distribution of \mathbf{X}_i . Without loss of generality, \mathbf{X}_i can be assumed to follow the correlation average,

$$\left\langle \sum_{i,j} \frac{\exp(is \cdot (\mathbf{X}_i - \mathbf{X}_j))}{V_c} \right\rangle = n_I(s), \quad (5)$$

where V_c is the volume of the region under consideration, and $n_I(s)$ is the static two-point correlation function of

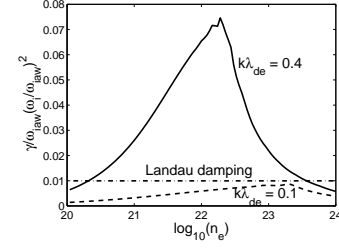


FIG. 1: The decay rates computed by the theory developed in this paper, compared with the prediction of the Landau damping theory (dot-dashed line) which is almost constant for a range of the wave vector. The parameters used are $m_i = 26$, $Z = 3$, $T_i = 20$ eV, and $T_e = 100$ eV. Two cases of the ion acoustic wave vectors are shown, $k = 0.1 k_{de}$ and $k = 0.4 k_{de}$. n_e is in the unit of cm^{-3} .

the ions. In the presence of spatially fixed ions, the electron's free wave eigenfunction is modified to be

$$|\sigma\rangle = |\mathbf{q}\rangle + \sum_i \int \frac{d^3 \mathbf{q}_1}{(2\pi)^3} \frac{\exp(-i\mathbf{q}_1 \cdot \mathbf{X}_i) U(\mathbf{q} - \mathbf{q}_1)}{E_{eff}(\mathbf{q}, \mathbf{q}_1)} |\mathbf{q}_1\rangle, \quad (6)$$

where $U(\mathbf{q}) = 4\pi Z e^2 / (|\mathbf{q}|^2 + k_{de}^2)$ is the Fourier transform of the screened ion-electron potential.

For the plasmon damping, $E_{eff}(\mathbf{q}, \mathbf{q}_1) = E(\mathbf{q}) - E(\mathbf{q}_1)$ can be used, as was used by Sturm [10]. This is a good approximation for the plasmons, as the plasmon energy $\hbar\omega_e$ is very high and the non-degenerate perturbation theory is good enough for the calculation of the distortion of the electron wave packet due to the presence of ions. However, in the case of the ion acoustic waves, the wave energy $\hbar\omega_{iaw}$ is much smaller than the electron temperature, and the perturbation theory is almost degenerate. In order to account for this effect, we choose $E_{eff}(\mathbf{q}, \mathbf{q}_1) = E(\mathbf{q}) + |E(\mathbf{q}) - E(\mathbf{q}_1)|$, which is a good approximation for the nearly elastic or large inelastic scattering. The susceptibility obtained from Eqs. (4), (5) and (6) is

$$\chi_e(\mathbf{k}, \omega) = \chi_{rpa}(\mathbf{k}, \omega) + \chi_{dense}(\mathbf{k}, \omega),$$

where χ_{rpa} is given in Eq. (1), and the subscript stands for the random phase approximation. χ_{dense} is of our main interest, which is given to be

$$\begin{aligned} \chi_{dense}(\mathbf{k}, \omega) &= \frac{4\pi Z e^2}{k^2} \int \frac{d^3 \mathbf{s}}{(2\pi)^3} n_I(\mathbf{s}) \\ &\times \int \frac{d^3 \mathbf{q}}{(2\pi)^3} \frac{U^2(\mathbf{s})}{A^2(\mathbf{q}, \mathbf{k}, \mathbf{s})} \frac{f(\mathbf{q} + \mathbf{k} + \mathbf{s}) - f(\mathbf{q})}{\hbar\omega - E(\mathbf{q} + \mathbf{k} + \mathbf{s}) + E(\mathbf{q})}, \end{aligned} \quad (7)$$

where $A(\mathbf{q}, \mathbf{k}, \mathbf{s})$ is

$$A^{-1}(\mathbf{q}, \mathbf{k}, \mathbf{s}) = \frac{1}{E_{eff}(\mathbf{q}, \mathbf{q} + \mathbf{s})} - \frac{1}{E_{eff}(\mathbf{q} + \mathbf{k}, \mathbf{q} + \mathbf{k} + \mathbf{s})}.$$

The imaginary part of χ_{dense} can be obtained from Eq. (7) by replacing the denominator, $1/(\hbar\omega - E(\mathbf{q} + \mathbf{k} + \mathbf{s}) + E(\mathbf{q}))$, by the delta function $\pi\delta(\hbar\omega - E(\mathbf{q} + \mathbf{k} + \mathbf{s}) + E(\mathbf{q}))$. For a Maxwellian plasma, $\text{Im}[\chi_{\text{dense}}]$ can be further simplified using the velocity integration $\mathbf{v} = \hbar\mathbf{q}/m_e$, instead of the wave vector \mathbf{q} :

$$\begin{aligned} \text{Im}[\chi_{\text{dense}}] = & \int \frac{n_I(\mathbf{s})d^3\mathbf{s}}{(2\pi)^3} \left(\frac{4\pi Ze^2}{k^2 + k_{\text{de}}^2} \right)^2 \left(\frac{\omega_e^2}{k^2} \right) \\ & \times \int d^3\mathbf{v} \left(A^{-2}(\mathbf{v}, \mathbf{k}, \mathbf{s}) \pi\delta(\omega - (\mathbf{k} + \mathbf{s}) \cdot \mathbf{v}) \right) \\ & \times \frac{f_M(\mathbf{v} + \frac{\hbar(\mathbf{k} + \mathbf{s})}{2m_e}) - f_M(\mathbf{v} - \frac{\hbar(\mathbf{k} + \mathbf{s})}{2m_e})}{\frac{\hbar}{m_e}}, \quad (8) \end{aligned}$$

where f_M is the Maxwellian distribution of the electron temperature T_e , satisfying the normalization condition $\int f_M d^3\mathbf{v} = 1$. $A^{-1}(\mathbf{v}, \mathbf{k}, \mathbf{s})$ is given to be

$$\begin{aligned} A^{-1}(\mathbf{v}, \mathbf{k}, \mathbf{s}) = & \frac{1}{E(\mathbf{v} - \frac{\mathbf{k} + \mathbf{s}}{2}) - |E(\mathbf{v} - \frac{\mathbf{k} + \mathbf{s}}{2}) - E(\mathbf{v} - \frac{\mathbf{k} - \mathbf{s}}{2})|} \\ & - \frac{1}{E(\mathbf{v} + \frac{\mathbf{k} - \mathbf{s}}{2}) - |E(\mathbf{v} + \frac{\mathbf{k} - \mathbf{s}}{2}) - E(\mathbf{v} + \frac{\mathbf{k} + \mathbf{s}}{2})|} \end{aligned}$$

where \mathbf{s} can be integrated in the spherical coordinate system. For a given set of \mathbf{k} and \mathbf{s} , the integration over the velocity can be reduced to a two-dimensional integral, as one variable is eliminated by the delta function. The decay rates given by Eqs. (3) and (8) (Fig. 1) exhibit that the newly computed decay rate is much higher than what is given by the Landau damping theory when $k\lambda_{\text{de}} = 0.4$. Similar integrations for various physical parameters show that the regime where the Umklapp process is important has the electron density ranging from 10^{21} to 10^{24} cm^{-3} .

The high ion charge would even further enhance the decay rate. In the presence of high-Z ions, the electrons interact more strongly with the ion acoustic wave, the ions being used as the momentum storage.

In summary, it is suggested that the ion acoustic wave decay rate could be much higher than the prediction by the Landau damping theory (see Eq. (8)). A rather rough theory, accounting for the effect of the Umklapp process on the decay, is presented. The theory proposed here is far from being complete. For instance, the perturbation expansion given in Eq. (6) may deviate significantly for the electrons of small kinetic energy (less than 1 eV). However, it highlights an important weak point of the prevalent Landau damping theory, when applied to dense plasmas: The ion acoustic wave damping in dense plasmas may arise from the three-party interaction (electron, ion, and wave), as opposed to the two-party interaction (electron and wave) the Landau damping theory is based on.

Some comments on the relationship of our theory to the plasma kinetic theory are in order. In plasma physics, the damping of various waves due to the ion-electron collisions is often considered in terms of the drag of the electron motion due to ions. For example, the inverse bremsstrahlung is treated in this way [14]. When the classical ion-electron collision picture breaks down, the dielectric function approach presented here is superior to the classical kinetic approach as demonstrated in the computation of the plasmon damping in the solid state physics problems [12, 13]. In the regime we consider, the classical electron-ion collisions are no longer valid and the quantum mechanical treatment is necessary. It would be interesting to consider the quantum ion acoustic wave as well [15], in the context of the quantum electron degeneracy and diffraction [16–18].

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